

Unitary-model-operator approach to Λ hypernuclei

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Abstract

A method is formulated for the description of lambda hypernuclei in the framework of the unitary-model-operator approach (UMOA). The method is applied to $^{17}_{\Lambda}\text{O}$. A lambda-nucleon effective interaction is derived, taking the coupling of the sigma-nucleon channel into account. The lambda single-particle energies are calculated for the $0s_{1/2}$, $0p_{3/2}$ and $0p_{1/2}$ states employing the Nijmegen soft-core (NSC), Jülich model- \tilde{A} ($J\tilde{A}$) and model- \tilde{B} ($J\tilde{B}$) hyperon-nucleon potentials.

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Unitary model operator approach; hypernuclei; lambda single particle energy; effective interaction; Nijmegen potential; Jülich potential

1 Introduction

There have been a number of attempts to study hypernuclei containing not only nucleons but also hyperons with strangeness [1–6]. It has been of high interest to investigate how the presence of strangeness broadens and/or modifies the knowledge achieved in the field of ordinary nuclear physics.

It is a fundamental problem to understand properties of a quantum many-body system in terms of basic interactions between constituent particles. In hypernuclear physics a unified treatment of all the baryon-baryon interactions, especially nucleon-nucleon (NN) and hyperon-nucleon (YN) interactions, will be necessary if one wish to understand general properties of hypernuclei. However, our knowledge on the YN interaction is quite inadequate in contrast to

that of the NN interaction due to experimental difficulties. In such a situation many-body theoretical studies of hypernuclei could provide alternative information on the YN interaction.

It has been discussed that physics of hypernuclei is different from conventional nuclear physics in some aspects. First, the channel couplings such as the ΣN - ΛN coupling play a significant role in the structure of hypernuclei in contrast to the very limited role of the ΔN - NN coupling appeared in nuclear many-body systems [7]. Second, the anti-symmetric spin-orbit force, which is forbidden in ordinary nuclei, emerges in hypernuclei in addition to the symmetric spin-orbit force.

The spin-orbit splitting of Λ single-particle levels in Λ hypernuclei was considered experimentally to be much smaller than their nucleonic counterpart [8]. However, some recent studies as in Ref. [9] suggest that the splitting would not be so small as discussed earlier. It will be highly desirable for the spin-orbit splitting to be established in experiment. The magnitude of the spin-orbit splitting will put further constraints on the YN interaction.

There have been several possibilities of solving the structure of hypernuclei. One possibility is to treat few-body or light hypernuclei by solving the Faddeev equation [10] or by applying the cluster model [11,12]. Another possibility is microscopic studies of hypernuclei with a larger mass number. While a few lower partial waves play a decisive role in few-body or light hypernuclei, higher partial waves could give some contributions in heavier hypernuclei. These microscopic studies of hypernuclei could make us understand interesting physics of their structure itself, and also help us in determining the YN interaction.

Microscopic structure calculation of heavier hypernuclei is usually performed by introducing a YN effective interaction. For this purpose the G -matrix theory has been applied in two ways: In some works an effective YN interaction has been derived by simulating a nuclear matter G -matrix. In this approach the non-locality and energy dependence of the G -matrix are neglected, and the effective YN interaction is represented in the two- or three-range Gaussian form with the adjustable Fermi momentum k_F . This type of the effective interaction has been referred to as the YNG interaction [13]. The YNG interaction has been successfully used in many of hypernuclear structure calculations [14]. It seems to us that the YNG interaction has made an important contribution to the considerable progress in overall description of hypernuclei. The other approach, in the framework of the G -matrix theory, is to calculate the G -matrix in finite hypernuclei by treating rigorously the Pauli exclusion operator and making a self-consistent calculation of single-particle potential of hyperons [15–19]. In contrast to studies of using the YNG interaction this approach possibly gives a better description of the state dependence of the effective YN interaction. It has been established that the G -matrix can be

used as a basic ingredient in constructing a microscopic many-body theory starting with bare interactions.

In spite of a marked success in the applications of the G -matrix theory, we may say from a formal point of view that the G -matrix is energy-dependent and non-hermitian, and does not have the property of decoupling between a model space of low-lying two-particle states and its complement. The G -matrix is not considered to be an effective interaction in a formal sense of the effective interaction theory. In order to derive an effective interaction we should add some higher-order corrections such as folded diagrams [20,21]. It would be desirable if we could have a theory describing many-body systems in terms of an energy-independent and hermitian effective interaction with the property of decoupling. Two of the present authors, K. S. and R. O., proposed a many-body theory, the unitary-model-operator approach (UMOA) that was formulated on the basis of such an effective interaction. The UMOA was applied to finite nuclei, ^{16}O and ^{40}Ca [22,23], and some improvements have been attained. The method may be viewed as an alternative way of deriving an effective interaction in finite many-body systems.

A unified study of hypernuclei and ordinary nuclei would be helpful in understanding many-body systems of baryons in a microscopic way. Therefore, we extended the formulation of the UMOA, in the previous work [24], to the description of hypernuclei and made a calculation of the properties in $^{17}_{\Lambda}\text{O}$ with a realistic YN interaction. The purposes of this work are to present a general formulation of the extended UMOA and apply it to $^{17}_{\Lambda}\text{O}$ by employing some realistic YN interactions.

This paper is organized as follows: In Section 2 we present a general formulation of the UMOA for deriving the ΛN effective interaction. In Section 3 the approximation procedure is given for making actual calculations. In Section 4 we apply the UMOA to $^{17}_{\Lambda}\text{O}$ by employing the Nijmegen soft-core (NSC) [25], Jülich model- \tilde{A} ($\tilde{J}\tilde{A}$) and model- \tilde{B} ($\tilde{J}\tilde{B}$) [26] YN potentials. In Section 5 we make some concluding remarks.

2 Formulation of the UMOA for the ΛN effective interaction

We present a formulation of the UMOA for applying to a calculation of the properties in Λ hypernuclei, especially, closed-shell nucleus plus one Λ systems. A main purpose is to give a method of calculating an effective interaction between Λ and N . In the derivation of the ΛN effective interaction we should note that, because of the small mass difference of about 77MeV between Λ and Σ , the coupling of the ΛN and ΣN channels plays an important role.

We first consider a two-body hamiltonian of a ΣN - ΛN coupled system given by

$$H_{YN} = h_{YN} + \Delta m + v_{YN}, \quad (2.1)$$

where

$$h_{YN} = \begin{pmatrix} t_\Lambda + u_\Lambda + t_N + u_N & 0 \\ 0 & t_\Sigma + u_\Sigma + t_N + u_N \end{pmatrix}, \quad (2.2)$$

$$\Delta m = \begin{pmatrix} 0 & 0 \\ 0 & m_\Sigma - m_\Lambda \end{pmatrix} \quad (2.3)$$

and

$$v_{YN} = \begin{pmatrix} v_{\Lambda N - \Lambda N} & v_{\Lambda N - \Sigma N} \\ v_{\Sigma N - \Lambda N} & v_{\Sigma N - \Sigma N} \end{pmatrix}. \quad (2.4)$$

The terms t_k and u_k for $k = \Lambda, \Sigma$ and N are the kinetic and single-particle potential energies of Λ, Σ and N , respectively, and v_{YN} is the bare two-body YN interaction including the ΛN and ΣN channels. The terms m_Λ and m_Σ are the rest masses of Λ and Σ , respectively. The hamiltonian H_{YN} describes a subsystem of interacting two particles ΛN or ΣN in the one-body potentials u_Λ , u_Σ and u_N .

In order to derive a ΛN effective interaction we introduce a unitarily transformed YN interaction given by

$$\tilde{v}_{YN} = e^{-S_{YN}}(h_{YN} + \Delta m + v_{YN})e^{S_{YN}} - (h_{YN} + \Delta m), \quad (2.5)$$

where S_{YN} is the correlation operator defined in a space of two-particle states of ΛN and ΣN . The S_{YN} is an anti-hermitian operator satisfying

$$S_{YN}^\dagger = -S_{YN}. \quad (2.6)$$

In the formulation of the UMOA the correlation operator S_{YN} is determined by the equation of decoupling for \tilde{v}_{YN} between a certain model space consisting of low-momentum two-particle states and its complement. This determination

of the correlation operator S_{YN} is one of the characteristics of the present approach.

For the determination of a ΛN effective interaction, we introduce a model space consisting of low-momentum ΛN states. Let $P_{\Lambda N}$ be the projection operator onto the model space. The complement of the $P_{\Lambda N}$ space includes a space of high-momentum ΛN states referred to as the $Q_{\Lambda N}$ space and a space of all the ΣN states referred to as the $Q_{\Sigma N}$ space. The condition of decoupling for the transformed interaction \tilde{v}_{YN} is given by

$$(Q_{\Lambda N} + Q_{\Sigma N})\tilde{v}_{YN}P_{\Lambda N} = P_{\Lambda N}\tilde{v}_{YN}(Q_{\Lambda N} + Q_{\Sigma N}) = 0. \quad (2.7)$$

Hereafter, we refer to the above equation as the decoupling equation. We assume that the one-body part $h_{YN} + \Delta m$ in Eq. (2.5) is decoupled as

$$(Q_{\Lambda N} + Q_{\Sigma N})(h_{YN} + \Delta m)P_{\Lambda N} = P_{\Lambda N}(h_{YN} + \Delta m)(Q_{\Lambda N} + Q_{\Sigma N}) = 0. \quad (2.8)$$

On the above assumption the decoupling equation (2.7) becomes

$$(Q_{\Lambda N} + Q_{\Sigma N})e^{-S_{YN}}(h_{YN} + \Delta m + v_{YN})e^{S_{YN}}P_{\Lambda N} = 0. \quad (2.9)$$

In general, the decoupling equation (2.9) does not determine S_{YN} uniquely. The usual restrictive conditions are

$$P_{\Lambda N}S_{YN}P_{\Lambda N} = (Q_{\Lambda N} + Q_{\Sigma N})S_{YN}(Q_{\Lambda N} + Q_{\Sigma N}) = 0. \quad (2.10)$$

These conditions are called the minimal effect requirements [28], which mean that the transformation $\exp(S_{YN})$ does not induce unnecessary transformation within each of the $P_{\Lambda N}$ and $Q_{\Lambda N} + Q_{\Sigma N}$ spaces.

In the general theory of constructing a hermitian effective interaction, the solution for S_{YN} in Eq. (2.9) has been known [29–31] and is given by

$$\begin{aligned} S_{YN} &= \text{arctanh}(\omega_{YN} - \omega_{YN}^\dagger) \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \{\omega_{YN}(\omega_{YN}^\dagger \omega_{YN})^n - \text{h.c.}\} \end{aligned} \quad (2.11)$$

with

$$\omega_{YN} = \sum_{k=1}^d (Q_{\Lambda N} + Q_{\Sigma N})|\Psi_{YNk}\rangle\langle\overline{\psi_{\Lambda Nk}}|P_{\Lambda N}, \quad (2.12)$$

where d is the dimension of the $P_{\Lambda N}$ space. The $|\Psi_{YNk}\rangle$ is the two-body eigenstate consisting of two components of ΛN and ΣN states, and satisfies

$$H_{YN}|\Psi_{YNk}\rangle = E_k|\Psi_{YNk}\rangle, \quad (2.13)$$

where

$$|\Psi_{YNk}\rangle = \begin{bmatrix} |\Psi_{\Lambda Nk}\rangle \\ |\Psi_{\Sigma Nk}\rangle \end{bmatrix}. \quad (2.14)$$

The $\langle \overline{\psi_{\Lambda Nk}}|$ in Eq. (2.12) is the biorthogonal state of $|\psi_{\Lambda Nk}\rangle$ satisfying

$$\langle \overline{\psi_{\Lambda Nk}}|\psi_{\Lambda Nk'}\rangle = \delta_{kk'}, \quad (2.15)$$

where $|\psi_{\Lambda Nk}\rangle$ is the $P_{\Lambda N}$ -space component of $|\Psi_{YNk}\rangle$ defined by

$$|\psi_{\Lambda Nk}\rangle = P_{\Lambda N}|\Psi_{YNk}\rangle. \quad (2.16)$$

It should be noted that the solution S_{YN} in Eq. (2.11) is determined dependently on the choice of a set of d eigenstates $\{|\Psi_{YNk}\rangle, k = 1, 2, \dots, d\}$ as given in Eq. (2.12). We choose a set of $\{|\Psi_{YNk}\rangle\}$ so that they have the largest $P_{\Lambda N}$ -space overlaps among all the eigenstates in Eq. (2.13). This is a usual choice in the derivation of the effective interaction.

With the solution S_{YN} in Eq. (2.11), the ΛN effective interaction in the $P_{\Lambda N}$ space is given by

$$\tilde{v}_{\Lambda N} = P_{\Lambda N}\tilde{v}_{YN}P_{\Lambda N}. \quad (2.17)$$

It has been known that this ΛN effective interaction can be written explicitly [22,32] as

$$\langle \phi_{\Lambda N\alpha}|\tilde{v}_{\Lambda N}|\phi_{\Lambda N\beta}\rangle = \frac{(1 + \mu_\alpha^2)^{\frac{1}{2}}\langle \phi_{\Lambda N\alpha}|R_{\Lambda N}|\phi_{\Lambda N\beta}\rangle + (1 + \mu_\beta^2)^{\frac{1}{2}}\langle \phi_{\Lambda N\alpha}|R_{\Lambda N}^\dagger|\phi_{\Lambda N\beta}\rangle}{(1 + \mu_\alpha^2)^{\frac{1}{2}} + (1 + \mu_\beta^2)^{\frac{1}{2}}}, \quad (2.18)$$

where $|\phi_{\Lambda N\alpha}\rangle$ ($|\phi_{\Lambda N\beta}\rangle$) and μ_α (μ_β) are defined through the eigenvalue equation

$$\omega_{YN}^\dagger \omega_{YN}|\phi_{\Lambda N\alpha}\rangle = \mu_\alpha^2|\phi_{\Lambda N\alpha}\rangle, \quad (2.19)$$

and $R_{\Lambda N}$ is given by

$$R_{AN} = P_{AN}(v_{YN} + v_{YN}\omega_{YN})P_{AN}. \quad (2.20)$$

The R_{AN} is a P_{AN} -space operator that agrees with the effective interaction of non-hermitian type. Equation (2.18) is a formula for converting the non-hermitian effective interaction to the hermitian one.

The term u_A in Eq. (2.2) is introduced as a self-consistent potential of A calculated with the AN effective interaction \tilde{v}_{AN} as

$$\langle \alpha_A | u_A | \alpha'_A \rangle = \sum_{\xi_N: \text{occupied}} \langle \alpha_A \xi_N | \tilde{v}_{AN} | \alpha'_A \xi_N \rangle, \quad (2.21)$$

where $|\alpha_A\rangle$ ($|\alpha'_A\rangle$) and $|\xi_N\rangle$ are the single-particle states of A and N , respectively.

From Eqs. (2.18)-(2.20) we see that the AN effective interaction \tilde{v}_{AN} can be calculated once the operator ω_{YN} is given. The operator ω_{YN} is determined completely, as given in Eq. (2.12), by solving the eigenvalue equation (2.13) and selecting d eigenstates $\{|\Psi_{YNk}\rangle\}$ with the largest P_{AN} -space overlaps. It is noted here that Eqs. (2.11)-(2.21) give a set of equations that determines the AN effective interaction \tilde{v}_{AN} self-consistently with the single-particle potential u_A .

3 Approximation procedure

3.1 Two-step method for the calculation of \tilde{v}_{YN}

Here we discuss a calculation procedure for obtaining the AN effective interaction acting in a small model space of low-lying AN states. We use the harmonic oscillator (h.o.) wave functions as basis states. It is desirable that the single-particle potentials of A , Σ and N in intermediate states are taken into account in a self-consistent way. However, as our main concern is to derive a AN effective interaction, the self-consistent treatment of u_A would have predominant importance in the present calculation. For this reason we make a self-consistent calculation only for u_A , and as for u_N we employ a fixed potential calculated previously for closed shell nuclei in the UMOA [22]. The effect of the single-particle potential u_Σ on the AN effective interaction has not necessarily been clarified, although some calculations with u_Σ have been made for the G -matrices in hyperonic nuclear matter [33,34]. In the actual calculations made so far the single-particle potential u_Σ has been considered to be rather shallow [14]. Although we have not known clearly how much the effect of u_Σ on the AN effective interaction, we here assume $u_\Sigma = 0$.

In solving the self-consistent equation for u_Λ with $\tilde{v}_{\Lambda N}$ we should be careful in the following two respects: (a) In the calculation of energies of intermediate states in propagators we should use u_Λ that is given for Λ states including high-momentum states. In order to do this we need a ΛN effective interaction acting in a large model space, because in general the $Q_{\Lambda N}$ space contains high-momentum ΛN states and the Q -space effective interaction $Q_{\Lambda N}\tilde{v}_{YN}Q_{\Lambda N}$ is not always well behaved. We thus make a calculation by introducing two model spaces, namely, large and small model spaces. The ΛN effective interaction in each of the model spaces is calculated by following a two-step procedure. In the first-step procedure, we define a large model space and calculate the ΛN effective interaction and single-particle potential of Λ in high-lying states. The ΛN effective interaction acting in this large model space will not be suitable for the effective interaction in a usual sense. By introducing a small model space and using the ΛN effective interaction determined in the first-step calculation, we proceed to the second-step calculation where we calculate a ΛN effective interaction acting in this small model space of low-lying ΛN states. (b) It is important in actual calculations how to determine the P and Q spaces because an effective interaction is determined dependently on the choice of the P and Q spaces. If a state in the P space mixes strongly with Q -space states in the eigenstate in Eq. (2.13), the matrix element of ω_{YN} or S_{YN} defined in Sec. 2 becomes large and as a result higher-order many-body correlations [35] give rise to non-negligible contributions. In general, the strong mixing takes place when some of the P - and Q -space states are quasi-degenerate in energy. For this reason, we should choose the P and Q spaces which are well separated in energy.

With due regard to points (a) and (b) we perform a calculation of the effective interaction through the two-step procedure. In the following subsections we present the two-step method in detail.

3.2 First-step decoupling

Let us write a two-particle state of Λ and N as the product of the h.o. single-particle wave functions in the usual notation as

$$|\alpha_{Y_i}\beta_N\rangle = |n_{Y_i}l_{Y_i}j_{Y_i}m_{Y_i}, n_Nl_Nj_Nm_N\rangle, \quad (Y_i = \Lambda, \Sigma). \quad (3.1)$$

We define a model space denoted by $P_{\Lambda N}^{(1)}$ and its complement $Q_{\Lambda N}^{(1)}$ with a boundary number ρ_1 as

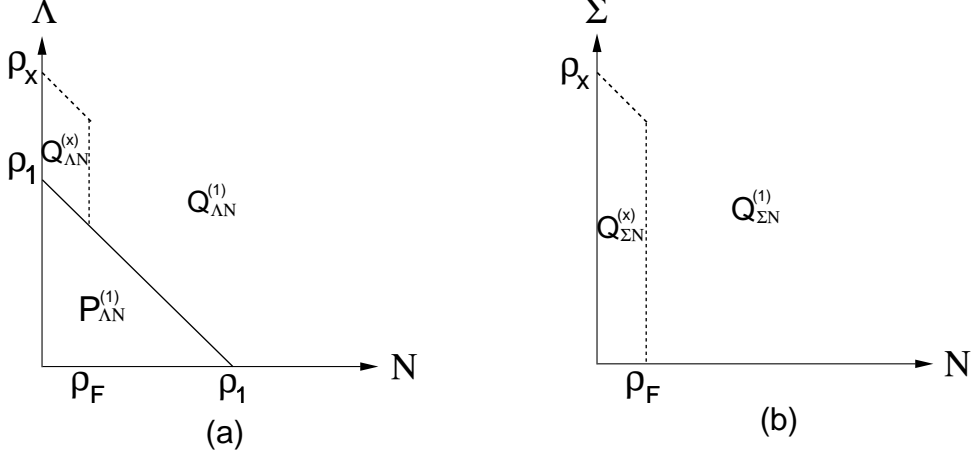


Fig. 1. The model space and its complement in the first-step calculation.

$$|\alpha_A \beta_N\rangle \in \begin{cases} P_{\Lambda N}^{(1)} & \text{if } 2n_A + l_A + 2n_N + l_N \leq \rho_1, \\ Q_{\Lambda N}^{(1)} & \text{otherwise.} \end{cases} \quad (3.2)$$

We also define a space of all the ΣN states denoted by $Q_{\Sigma N}^{(1)}$ as

$$|\alpha_\Sigma \beta_N\rangle \in Q_{\Sigma N}^{(1)}. \quad (3.3)$$

Note that the model space $P_{\Lambda N}^{(1)}$ does not contain the ΣN states.

If we make a graph of two-particle states, the $P_{\Lambda N}^{(1)}$, $Q_{\Lambda N}^{(1)}$ and $Q_{\Sigma N}^{(1)}$ spaces are represented in Fig. 1. The ρ_1 means a number such that $(\rho_1 + 3)\hbar\Omega/2$ gives the maximum kinetic energy of the ΛN two-particle states in the $P_{\Lambda N}^{(1)}$ space. The $\hbar\Omega$ denotes the unit of the h.o. energy which we use commonly for Λ , Σ and N . As we require to introduce a sufficiently large model space in the first-step calculation, we should choose ρ_1 as large as possible. Another requirement is that the states in the $P_{\Lambda N}^{(1)}$ space should be separated in energy from the states in the $Q_{\Lambda N}^{(1)}$ space. For this reason, we adopt a model space $P_{\Lambda N}^{(1)}$ of a triangular shape. This choice of the $P_{\Lambda N}^{(1)}$ space guarantees the separation in energy.

In Fig. 1 the spaces $Q_{Y_i N}^{(x)}$ for $Y_i = \Lambda$ and Σ , specified by the numbers ρ_1, ρ_F and ρ_x , stand for the excluded spaces due to the Pauli principle for nucleons. The number ρ_F is defined as $\rho_F = 2n_N + l_N$ with the quantum numbers $\{n_N, l_N\}$ of the highest occupied orbit in the closed-shell core nucleus. The ρ_x means a number such that $(\rho_x + 3)\hbar\Omega/2$ gives the maximum kinetic energy of the YN two-particle states in the $Q_{\Lambda N}^{(x)}$ and $Q_{\Sigma N}^{(x)}$ spaces. We then write the $Q_{\Lambda N}^{(x)}$ and $Q_{\Sigma N}^{(x)}$ spaces as

$$|\alpha_A \beta_N\rangle \in Q_{\Lambda N}^{(x)} \quad \text{if } \rho_1 < 2n_A + l_A + 2n_N + l_N \leq \rho_x \\ \text{and } 0 \leq 2n_N + l_N \leq \rho_F \quad (3.4)$$

and

$$\begin{aligned} |\alpha_{\Sigma}\beta_N\rangle \in Q_{\Sigma N}^{(X)} \quad & \text{if } 0 \leq 2n_{\Sigma} + l_{\Sigma} + 2n_N + l_N \leq \rho_X \\ & \text{and } 0 \leq 2n_N + l_N \leq \rho_F. \end{aligned} \quad (3.5)$$

We also define the sum of the excluded spaces as

$$Q_{YN}^{(X)} \equiv Q_{AN}^{(X)} + Q_{\Sigma N}^{(X)}. \quad (3.6)$$

Furthermore, by using the projection operators, $Q_{AN}^{(1)}$, $Q_{AN}^{(X)}$, $Q_{\Sigma N}^{(1)}$ and $Q_{\Sigma N}^{(X)}$, we introduce projection operators onto the spaces of allowed YN states as

$$\overline{Q}_{AN}^{(1)} \equiv Q_{AN}^{(1)} - Q_{AN}^{(X)}, \quad (3.7)$$

$$\overline{Q}_{\Sigma N}^{(1)} \equiv Q_{\Sigma N}^{(1)} - Q_{\Sigma N}^{(X)}, \quad (3.8)$$

and we also define

$$\overline{Q}_{YN}^{(1)} \equiv \overline{Q}_{AN}^{(1)} + \overline{Q}_{\Sigma N}^{(1)}. \quad (3.9)$$

Considering the Pauli-exclusion principle for nucleons the terms $Q_{YN}^{(X)}v_{YN}Q_{YN}^{(X)}$ and $P_{AN}^{(1)}v_{YN}Q_{YN}^{(X)} + \text{h.c.}$ should be removed in treating the problem of two-body correlations. However, if we remove all the matrix elements of these terms, this leads to overmuch counting of the Pauli principle. The matrix elements which are diagonal in nucleon states should be restored even though nucleons are in occupied states. Therefore, instead of v_{YN} , we should use a YN interaction \overline{v}_{YN} defined as

$$\overline{v}_{YN} \equiv (P_{AN}^{(1)} + \overline{Q}_{YN}^{(1)})v_{YN}(P_{AN}^{(1)} + \overline{Q}_{YN}^{(1)}) + \sum_{\nu_N} \sum_{\substack{Y_i=A,\Sigma \\ Y_j=A,\Sigma}} Q_{Y_i N}^{(d)}(\nu_N)v_{YN}Q_{Y_j N}^{(d)}(\nu_N), \quad (3.10)$$

where ν_N denotes the set of the h.o. quantum numbers $\{n_N, l_N\}$ of an occupied state of a nucleon, and the operator $Q_{Y_i N}^{(d)}(\nu_N)$ is defined as

$$Q_{Y_i N}^{(d)}(\nu_N) \equiv \sum_{\alpha_{Y_i}} |\alpha_{Y_i}\nu_N\rangle \langle \alpha_{Y_i}\nu_N|, \quad (Y_i = A, \Sigma). \quad (3.11)$$

The two-body YN equation to be solved in the first-step calculation is now written as

$$(h_{YN}^{(1)} + \Delta m + \bar{v}_{YN})|YN; k_1\rangle = E_{k_1}|YN; k_1\rangle, \quad (3.12)$$

where we define $h_{YN}^{(1)}$ as

$$h_{YN}^{(1)} = \begin{pmatrix} t_A + u_A^{(1)} + t_N + u_N & 0 \\ 0 & t_\Sigma + t_N + u_N \end{pmatrix}. \quad (3.13)$$

The $u_A^{(1)}$ is the single-particle potential of A to be calculated with the AN effective interaction self-consistently. The number k_1 stands for a set of quantum numbers to specify a two-body YN eigenstate. The unitarily transformed YN interaction in the first-step calculation is given by

$$\tilde{v}_{YN}^{(1)} = e^{-S_{YN}^{(1)}}(h_{YN}^{(1)} + \Delta m + \bar{v}_{YN})e^{S_{YN}^{(1)}} - (h_{YN}^{(1)} + \Delta m). \quad (3.14)$$

The decoupling equation for determining the correlation operator $S_{YN}^{(1)}$ becomes

$$\bar{Q}_{YN}^{(1)} \tilde{v}_{YN}^{(1)} P_{AN}^{(1)} = P_{AN}^{(1)} \tilde{v}_{YN}^{(1)} \bar{Q}_{YN}^{(1)} = 0. \quad (3.15)$$

The AN effective interaction $\tilde{v}_{YN}^{(1)}$ in the $P_{AN}^{(1)}$ space is then given by

$$\tilde{v}_{AN}^{(1)} = P_{AN}^{(1)} \tilde{v}_{YN}^{(1)} P_{AN}^{(1)}. \quad (3.16)$$

In the first-step procedure we solve the decoupling equation (3.15) by introducing some assumptions. As we take a sufficiently large number as ρ_1 , we may assume that the one-body potential of A for a state in the $Q_{AN}^{(1)}$ space is negligible in comparison with the sum of the energies, $t_A + t_N + u_N$. Therefore, $u_A^{(1)}$ takes nonzero values only for states in the $P_{AN}^{(1)}$ space, and we have

$$u_A^{(1)} = P_{AN}^{(1)} u_A^{(1)} P_{AN}^{(1)}. \quad (3.17)$$

The definitions of $P_{AN}^{(1)}$ and $Q_{AN}^{(1)}$ in Eq. (3.2) allow us to divide a space of AN states written in relative and center-of-mass (c.m.) coordinates into the same subspaces $P_{AN}^{(1)}$ and $Q_{AN}^{(1)}$ as

$$|AN; nlS J_r, N_c L_c\rangle \in \begin{cases} P_{AN}^{(1)} & \text{if } 2n + l + 2N_c + L_c \leq \rho_1, \\ Q_{AN}^{(1)} & \text{otherwise,} \end{cases} \quad (3.18)$$

where $\{n, l\}$ and $\{N_c, L_c\}$ are the h.o. quantum numbers of relative and c.m. motions, respectively. The S and J_r denote, respectively, spin and angular momentum defined through $\vec{J}_r = \vec{l} + \vec{S}$. The isospin T is omitted because T takes a constant $1/2$ in the ΛN system. A ΣN state in the $Q_{\Sigma N}^{(1)}$ space in Fig. 1 is also written by the product of the h.o. relative and c.m. states.

The YN interaction \bar{v}_{YN} in Eq. (3.10) is not diagonal in the c.m. quantum numbers $\{N_c, L_c\}$ because the projection operators $P_{\Lambda N}^{(1)}$, $\bar{Q}_{YN}^{(1)}$ and $Q_{Y_i N}^{(d)}$ are not diagonal in $\{N_c, L_c\}$. Thus, the ΛN effective interaction $\tilde{v}_{\Lambda N}^{(1)}$ is not diagonal in $\{N_c, L_c\}$. It has been shown, however, in the calculation of ordinary nuclei that $\{N_c, L_c\}$ non-diagonal elements of the G -matrix are generally small [36]. Therefore, we make an assumption that the $\{N_c, L_c\}$ non-diagonal matrix elements of $\tilde{v}_{\Lambda N}^{(1)}$ can be neglected.

The two-body eigenvalue equation (3.12) is solved for the relative motion with a fixed set of $\{N_c, L_c\}$ written by

$$(h_{YN}^{(1)} + \Delta m + \bar{v}_{YN})|YN; k_1, ll'SS'J_r, N_c L_c\rangle = E_{k_1}|YN; k_1, ll'SS'J_r, N_c L_c\rangle, \quad (3.19)$$

where we rewrite $h_{YN}^{(1)}$ as

$$h_{YN}^{(1)} = \begin{pmatrix} t_{r\Lambda N} + t_{c\Lambda N} + u_A^{(1)} + u_N & 0 \\ 0 & t_{r\Sigma N} + t_{c\Sigma N} + u_N \end{pmatrix}. \quad (3.20)$$

The t_{rYN} and t_{cYN} are the kinetic energies of the YN relative and c.m. motions, respectively. The state $|YN; k_1, ll'SS'J_r, N_c L_c\rangle$ expresses a superposition of the ΛN and ΣN h.o. states and gives a general form of the eigenstate when we consider the ΣN - ΛN coupling and the YN interaction with the tensor and antisymmetric spin-orbit couplings. The k_1 is an additional quantum number specifying a two-body YN eigenstate.

In general, the one-body part $h_{YN}^{(1)} + \Delta m$ does not satisfies the assumption of decoupling in Eq. (2.8). However, it has been shown in the calculation in nuclei that if we take a larger number as ρ_1 , the effect of violating the assumption of decoupling becomes considerably smaller [22,23]. We have confirmed that this non-decoupling effect becomes negligible in calculating nuclear effective interactions. We here assume this to be the case also in the determination of the ΛN effective interaction. Furthermore, we introduce another approximation that the sum of $t_{cY_i N}$ and $u_{Y_i} + u_N$ be diagonal in the c.m. quantum numbers N_c and L_c . This assumption has also been made sure to be acceptable in the previous works [22–24].

In order to solve the eigenvalue equation (3.19) we need to calculate the ma-

trix elements of $u_{Y_i}^{(1)} + u_N$ in the relative and c.m. coordinates representation. However, the calculation of these matrix elements is difficult because the term $u_{Y_i}^{(1)} + u_N$ can not be separated into a sum of operators written in relative and c.m. coordinates. Therefore, as have made in the previous work [22], we employ the angle-average approximation and have

$$\begin{aligned}
& \langle Y_i N; nl S J_r, N_c L_c | u_{Y_i}^{(1)} + u_N | Y_i N; n' l' S' J_r, N_c L_c \rangle \\
&= \sum_{\substack{n_{Y_i} l_{Y_i} n_N l_N \\ n'_{Y_i} n'_N \lambda}} \frac{2\lambda + 1}{(2l + 1)(2L_c + 1)} \langle nl N_c L_c : \lambda | n_{Y_i} l_{Y_i} n_N l_N : \lambda \rangle_{Y_i N} \\
&\times (\langle n' l' N_c L_c : \lambda | n'_{Y_i} l_{Y_i} n'_N l_N : \lambda \rangle_{Y_i N} \langle n_{Y_i} l_{Y_i} | u_{Y_i}^{(1)} | n'_{Y_i} l_{Y_i} \rangle \\
&\quad + \langle n' l' N_c L_c : \lambda | n_{Y_i} l_{Y_i} n'_N l_N : \lambda \rangle_{Y_i N} \langle n_N l_N | u_N | n'_N l_N \rangle) \delta_{ll'} \delta_{SS'}, (Y_i = \Lambda, \Sigma),
\end{aligned} \tag{3.21}$$

where, as was assumed previously, $u_{Y_i}^{(1)} = 0$ for $Y_i = \Sigma$. The $\langle nl N_c L_c : \lambda | n_{Y_i} l_{Y_i} n_N l_N : \lambda \rangle_{Y_i N}$ is the h.o. transformation bracket [37] for the YN two-body state which depends on the rest masses for $Y_i = \Lambda$ and Σ . Here the angle-average approximation means that, when a function $f(jm)$ is given with an angular momentum j and its z -component m , we approximate $f(jm)$ as $\bar{f}(j) = \sum_m f(jm)/(2j + 1)$. In the present case the matrix element on the left-hand side in Eq. (3.21) is dependent on the z -components m_{J_r} and m_{L_c} of J_r and L_c , respectively. The average over m_{J_r} and m_{L_c} leads to the expression given in Eq. (3.21). This approximation is essentially the same as has been made to derive an angle-averaged Pauli operator by Wong and Sauer [38,39].

The use of the basis states $\{| Y_i N; nl S J_r, N_c L_c \rangle\}$ makes it difficult to represent the operator $\bar{Q}_{YN}^{(1)}$, but it has been confirmed that the angle-average technique works well. We thus make an approximation to the operator $\bar{Q}_{YN}^{(1)}$ as

$$\bar{Q}_{YN}^{(1)} \simeq \sum_{\substack{nl N_c L_c S J_r \\ Y_i = \Lambda, \Sigma}} \theta_{Y_i N}(nl, N_c L_c) | Y_i N; nl S J_r, N_c L_c \rangle \langle Y_i N; nl S J_r, N_c L_c |, \tag{3.22}$$

where the summation should be made on the conditions

$$\begin{aligned}
\rho_1 &< 2n + l + 2N_c + L_c \leq \rho_x \text{ for } Y_i = \Lambda, \\
0 &\leq 2n + l + 2N_c + L_c \leq \rho_x \text{ for } Y_i = \Sigma.
\end{aligned} \tag{3.23}$$

Here the coefficient $\theta_{Y_i N}(nl, N_c L_c)$ is defined as

$$\theta_{Y_i N}(nl, N_c L_c)$$

$$= 1 - \sum_{n_{Y_i} l_{Y_i} n_N l_N \lambda} \frac{(2\lambda + 1)}{(2l + 1)(2L_c + 1)} \langle nl N_c L_c : \lambda | n_{Y_i} l_{Y_i} n_N l_N : \lambda \rangle_{Y_i N}^2, \quad (Y_i = A, \Sigma), \quad (3.24)$$

where in the summation the following conditions should be satisfied

$$\begin{aligned} 0 &\leq 2n_N + l_N \leq \rho_F, \\ \rho_1 &< 2n_{Y_i} + l_{Y_i} + 2n_N + l_N \leq \rho_X \text{ for } Y_i = A, \\ 0 &\leq 2n_{Y_i} + l_{Y_i} + 2n_N + l_N \leq \rho_X \text{ for } Y_i = \Sigma. \end{aligned} \quad (3.25)$$

Furthermore, we need an expression of $Q_{YN}^{(d)}(\nu_N)$ given in Eq. (3.11) in the angle-average approximation

$$\begin{aligned} Q_{YN}^{(d)}(\nu_N) &\simeq \sum_{nl N_c L_c S J_r} \theta'_{Y_i N}(nl, N_c L_c, n_N l_N) |Y_i N; nl S J_r, N_c L_c\rangle \langle Y_i N; nl S J_r, N_c L_c|, \\ &\quad (Y_i = A, \Sigma), \end{aligned} \quad (3.26)$$

where the summation should be made on the same conditions as in Eq. (3.23). The coefficient $\theta'_{Y_i N}(nl, N_c L_c, n_N l_N)$ is given by

$$\begin{aligned} &\theta'_{Y_i N}(nl, N_c L_c, n_N l_N) \\ &= \sum_{n_{Y_i} l_{Y_i} \lambda} \frac{(2\lambda + 1)}{(2l + 1)(2L_c + 1)} \langle nl N_c L_c : \lambda | n_{Y_i} l_{Y_i} n_N l_N : \lambda \rangle_{Y_i N}^2, \quad (Y_i = A, \Sigma), \end{aligned} \quad (3.27)$$

where n_{Y_i} and l_{Y_i} should satisfy the conditions given in Eq. (3.25).

Once the set of the eigenstates $|YN; k_1, ll' SS' J_r, N_c L_c\rangle$ in Eq. (3.19) is solved, the AN effective interaction $\tilde{v}_{AN}^{(1)}$ acting in the $P_{AN}^{(1)}$ space is given according to Eqs. (2.18)-(2.20) by substituting $\omega_{YN} = \omega_{YN}^{(1)}$, where $\omega_{YN}^{(1)}$ is defined as

$$\omega_{YN}^{(1)} = \sum_{k_1, ll' SS' J_r, N_c L_c} \overline{Q}_{YN}^{(1)} |YN; k_1, ll' SS' J_r, N_c L_c\rangle \langle \overline{YN}; k_1, ll' SS' J_r, N_c L_c| P_{AN}^{(1)}. \quad (3.28)$$

With the AN effective interaction $\tilde{v}_{AN}^{(1)}$ thus determined, we calculate a new single-particle potential $\langle n_A l_A | u_A^{(1)} | n'_A l_A \rangle$ as an averaged matrix element of $\{\langle n_A l_A j_A | u_A^{(1)} | n'_A l_A j_A \rangle, j_A = l_A \pm 1/2\}$. The $\langle n_A l_A | u_A^{(1)} | n'_A l_A \rangle$ can be given explicitly in terms of the effective interaction $\tilde{v}_{AN}^{(1)}$ as

$$\begin{aligned}
\langle n_A l_A | u_A^{(1)} | n'_A l_A \rangle &\equiv \frac{\sum_{j_A} (2j_A + 1) \langle n_A l_A j_A | u_A^{(1)} | n'_A l_A j_A \rangle}{\sum_{j_A=l_A \pm \frac{1}{2}} (2j_A + 1)} \\
&= \sum_{\substack{nn'lSJ_r N_c L_c \lambda \\ \{n_N, l_N\}: \text{occupied}}} \frac{(2\lambda + 1)(2J_r + 1)}{(2l_A + 1)(2l + 1)} \\
&\quad \times \langle nl N_c L_c : \lambda | n_A l_A n_N l_N : \lambda \rangle_{AN} \langle n' l N_c L_c : \lambda | n'_A l_A n_N l_N : \lambda \rangle_{AN} \\
&\quad \times \langle AN; nl S J_r, N_c L_c | \tilde{v}_{AN}^{(1)} | AN; n' l S J_r, N_c L_c \rangle. \tag{3.29}
\end{aligned}$$

In the above treatment of the single-particle potential of Λ in the first-step calculation, we do not consider the one-body spin-orbit splitting.

The ΛN effective interaction $\tilde{v}_{AN}^{(1)}$ in Eq. (3.16) is dependent on the single-particle potential $u_A^{(1)}$, and $u_A^{(1)}$ is determined with $\tilde{v}_{AN}^{(1)}$ as in Eq. (3.29). Therefore, the equation for determining $\tilde{v}_{AN}^{(1)}$ and $u_A^{(1)}$ should be solved simultaneously.

3.3 Second-step decoupling

The model space $P_{AN}^{(1)}$ in the first-step procedure is chosen as a *low-momentum* space, but the $P_{AN}^{(1)}$ space is still large when we wish to use $\tilde{v}_{AN}^{(1)}$ in the calculation of bound or low excited states of Λ hypernuclei. With $\tilde{v}_{AN}^{(1)}$, we further proceed to the calculation of the ΛN effective interaction acting in a smaller model space and correspondingly the single-particle potential of Λ in low-lying states. We separate the $P_{AN}^{(1)}$ space into the $P_{AN}^{(2)}$, $P_{AN}^{(X)}$, $P_{AN}^{(Y)}$ and $Q_{AN}^{(2)}$ spaces specified by the numbers ρ_1 , ρ_F , and ρ_2 as shown in Fig. 2. The number ρ_2 is introduced to specify the uppermost bound state of Λ . Here these spaces are defined as

$$\begin{aligned}
&|\alpha_A \beta_N \rangle \\
&\in \begin{cases} P_{AN}^{(2)} & \text{if } 2n_A + l_A \leq \rho_2 \text{ and } 2n_N + l_N \leq \rho_F, \\ P_{AN}^{(X)} & \text{if } 2n_A + l_A > \rho_2, \ 2n_N + l_N \leq \rho_F \text{ and } 2n_A + l_A + 2n_N + l_N \leq \rho_1, \\ P_{AN}^{(Y)} & \text{if } 2n_A + l_A \leq \rho_2, \ 2n_N + l_N > \rho_F \text{ and } 2n_A + l_A + 2n_N + l_N \leq \rho_1, \\ Q_{AN}^{(2)} & \text{if } 2n_A + l_A > \rho_2, \ 2n_N + l_N > \rho_F \text{ and } 2n_A + l_A + 2n_N + l_N \leq \rho_1. \end{cases} \tag{3.30}
\end{aligned}$$

The $P_{AN}^{(X)}$ space is the excluded space due to the Pauli principle for nucleons. We are principally interested in the ΛN effective interaction that gives a description of the bound states of Λ . It would be desirable that such a ΛN effective interaction satisfies the condition of decoupling between two ΛN states consisting of the bound and unbound states of Λ . This requirement is equivalent to the situation that the ΛN effective interactions should be decoupled

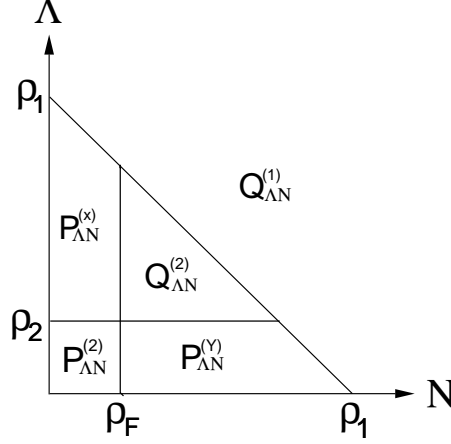


Fig. 2. The model space and its complement in the second-step calculation.

between two ΛN states in the $P_{\Lambda N}^{(2)}$ and $Q_{\Lambda N}^{(2)}$ spaces. Therefore, we solve the decoupling equation for $\tilde{v}_{\Lambda N}^{(2)}$ between the ΛN states in the $P_{\Lambda N}^{(2)}$ and $Q_{\Lambda N}^{(2)}$ spaces.

The space $P_{\Lambda N}^{(Y)}$ is not a space to be excluded due to the Pauli principle. However, some states in the $P_{\Lambda N}^{(Y)}$ and $P_{\Lambda N}^{(2)}$ spaces overlap each other in energy. Therefore, we treat separately the effect of the coupling between the $P_{\Lambda N}^{(Y)}$ and $P_{\Lambda N}^{(2)}$ spaces. Actually the interaction between states in the $P_{\Lambda N}^{(Y)}$ and $P_{\Lambda N}^{(2)}$ spaces induces the core polarization in the core nucleus, as discussed in Refs. [22] and [24]. We take into account the effect of the core polarization in the usual perturbative calculation.

In the second-step calculation the unitarily transformed interaction is given by

$$\tilde{v}_{\Lambda N}^{(2)} = e^{-S_{\Lambda N}^{(2)}}(h_{\Lambda N}^{(2)} + \tilde{v}_{\Lambda N}^{(1)})e^{S_{\Lambda N}^{(2)}} - h_{\Lambda N}^{(2)}, \quad (3.31)$$

where

$$h_{\Lambda N}^{(2)} = t_A + u_A^{(2)} + t_N + u_N. \quad (3.32)$$

The term $u_A^{(2)}$ is the self-consistent potential of Λ to be determined in the second-step procedure. The decoupling equation for determining the correlation operator $S_{\Lambda N}^{(2)}$ in Eq. (3.31) becomes

$$Q_{\Lambda N}^{(2)} \tilde{v}_{\Lambda N}^{(2)} P_{\Lambda N}^{(2)} = P_{\Lambda N}^{(2)} \tilde{v}_{\Lambda N}^{(2)} Q_{\Lambda N}^{(2)} = 0. \quad (3.33)$$

We note here that the assumption of decoupling

$$Q_{\Lambda N}^{(2)} h_{\Lambda N}^{(2)} P_{\Lambda N}^{(2)} = 0 \quad (3.34)$$

is satisfied exactly because the two two-body states in the $P_{AN}^{(2)}$ and $Q_{AN}^{(2)}$ spaces do not contain a common single-particle state of Λ or N .

The two-body eigenvalue equation to be solved in the second-step calculation is written as

$$(P_{AN}^{(2)} + Q_{AN}^{(2)})(h_{AN}^{(2)} + \tilde{v}_{AN}^{(1)})(P_{AN}^{(2)} + Q_{AN}^{(2)})|\Lambda N; k_2, J\rangle = E_{k_2}|\Lambda N; k_2, J\rangle, \quad (3.35)$$

where J denotes the total angular momentum, and k_2 a set of quantum numbers of a ΛN two-body eigenstate. It should be noted that this eigenvalue equation can be solved exactly in the $P_{AN}^{(2)} + Q_{AN}^{(2)}$ space by the diagonalization of the hamiltonian written with the basis states $\{|n_\Lambda l_\Lambda j_\Lambda, n_N l_N j_N; JM\rangle\}$ if the $P_{AN}^{(2)} + Q_{AN}^{(2)}$ space is chosen suitably as a small space. Once a set of the eigenstates $|\Lambda N; k_2, J\rangle$ in Eq. (3.35) is solved, the ΛN effective interaction $\tilde{v}_{AN}^{(2)}$ in the $P_{AN}^{(2)}$ space is given from Eqs. (2.18)-(2.20) with $\omega_{YN} = \omega_{AN}^{(2)}$ defined by

$$\omega_{AN}^{(2)} = \sum_{k_2, J} Q_{AN}^{(2)}|\Lambda N; k_2, J\rangle\langle\Lambda N; k_2, J|P_{AN}^{(2)}. \quad (3.36)$$

The single-particle potential $u_A^{(2)}$ is calculated with the ΛN effective interactions, $\tilde{v}_{AN}^{(2)}$ and $\tilde{v}_{AN}^{(1)}$, as

$$\begin{aligned} & \langle\alpha_A|u_A^{(2)}|\alpha'_A\rangle \\ &= \sum_{\beta_N: \text{occupied}} \begin{cases} \langle\alpha_A\beta_N|\tilde{v}_{AN}^{(2)}|\alpha'_A\beta_N\rangle & \text{if } |\alpha_A\beta_N\rangle \in P_{AN}^{(2)} \text{ and } |\alpha'_A\beta_N\rangle \in P_{AN}^{(2)}, \\ \langle\alpha_A\beta_N|\tilde{v}_{AN}^{(1)}|\alpha'_A\beta_N\rangle & \text{otherwise.} \end{cases} \end{aligned} \quad (3.37)$$

We solve the set of equations for determining $\tilde{v}_{AN}^{(2)}$ and $u_A^{(2)}$ iteratively until the result converges.

We remark here that we employ $\tilde{v}_{AN}^{(1)}$ as the effective interaction in the $P_{AN}^{(X)}$ space in the calculation of $u_A^{(2)}$. Since the effective interaction in the $P_{AN}^{(X)}$ space is unchanged in the transformation made in the second-step procedure, we may write the ΛN effective interaction in the $P_{AN}^{(X)}$ space as $P_{AN}^{(X)}\tilde{v}_{AN}^{(2)}P_{AN}^{(X)} = P_{AN}^{(X)}\tilde{v}_{AN}^{(1)}P_{AN}^{(X)}$. We, therefore, have used partly the ΛN effective interaction $\tilde{v}_{AN}^{(1)}$ in the calculation of $u_A^{(2)}$ as in Eq. (3.37).

3.4 Corrections in perturbative treatment

First we note that the one-body hamiltonian $h_{AN}^{(2)}$ in Eq. (3.32) has non-diagonal terms because we did not make the Hartree-Fock calculation. These

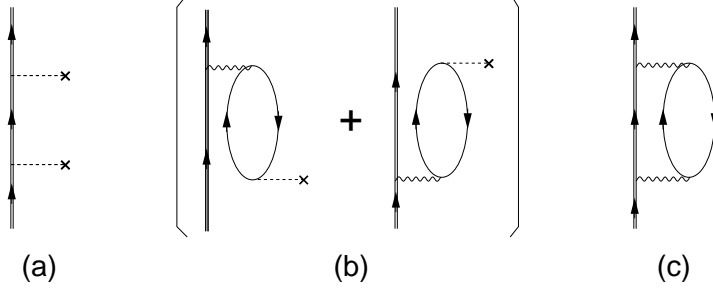


Fig. 3. Calculated diagrams for the Λ single-particle energies. The ΛN effective interaction $\tilde{v}_{\Lambda N}$ is represented by the wavy-line vertex, and the \times vertex with the dashed line represents the non-diagonal part of the one-body hamiltonian $h_k = t_k + u_k$ for $k = \Lambda$ and N . The propagations of N and Λ are represented by the single and double external lines, respectively.

terms could bring about sizable corrections to the single-particle energies of Λ . We adopt the same h.o. energy both for Λ and N , though it has been considered that the wave function of Λ is basically different from that of N . Therefore, we must evaluate the corrections arising from the non-diagonal terms of $h_{\Lambda N}^{(2)}$. By taking the diagonal part of $h_{\Lambda N}^{(2)}$ as the unperturbed hamiltonian we calculate these corrections perturbatively up to second order as shown in diagram (a) of Fig. 3.

Next, the interaction between the ΛN states in the $P_{\Lambda N}^{(2)}$ and $P_{\Lambda N}^{(Y)}$ spaces induces the core polarization in the core nucleus, and is important in the calculation of the corrections. We here assume that these corrections can be calculated approximately, using the interaction $P_{\Lambda N}^{(2)}\tilde{v}_{\Lambda N}^{(1)}P_{\Lambda N}^{(Y)} + \text{h.c.}$. It will be desirable to take into account the effect of intermediate nucleon states with high momentum. In this perturbative calculation we include only the nucleon states belonging to the $P_{\Lambda N}^{(Y)}$ space, that is, the nucleon h.o. state should satisfy $2n_N + l_N \leq \rho_1 - (2n_A + l_A)$, where $\{n_A, l_A\}$ are the h.o. quantum numbers of a Λ state. The validity of this truncation of nucleon states will be checked in the numerical calculation in the next section by increasing the number ρ_1 . The diagrams corresponding these corrections are given in diagram (b) and (c) of Fig. 3.

4 Application to ${}^{17}_\Lambda\text{O}$

We performed calculations employing the Jülich model- \tilde{A} ($\tilde{J}\tilde{A}$), model- \tilde{B} ($\tilde{J}\tilde{B}$) [26]¹ and Nijmegen soft-core (NSC) [25]² potentials for the YN interaction.

¹ We used the computer code for the Jülich YN potentials in the momentum-space representation made by B. Holzenkamp and A. Reuber.

² We used the computer code for the Nijmegen soft-core YN potential in the coordinate-space representation provided by T. A. Rijken and J. J. de Swart.

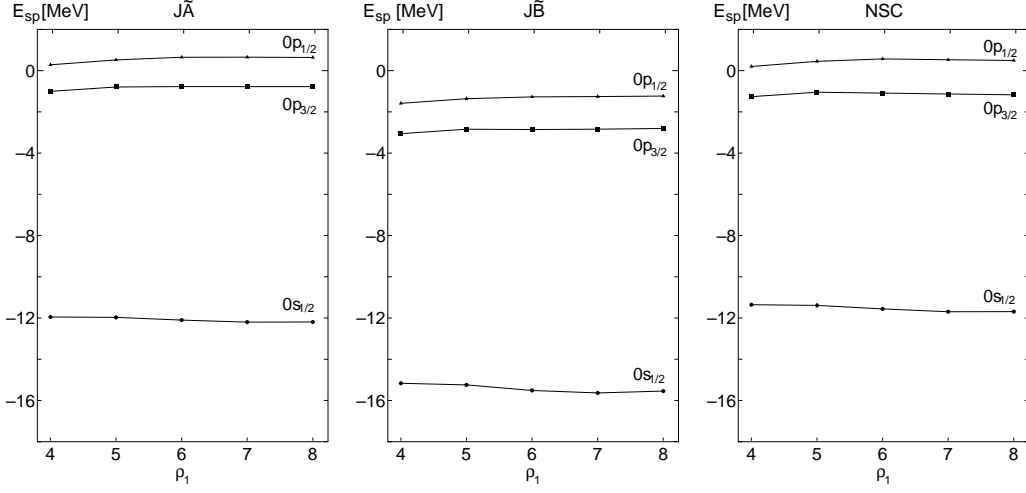


Fig. 4. The ρ_1 dependence of the calculated Λ single-particle energies including the perturbation corrections given in Fig. 3.

We used the h.o. basis states with a common $\hbar\Omega = 14\text{MeV}$ for Λ , Σ and N . The dependence of the effective interaction and single-particle energies on the value $\hbar\Omega$ has been already discussed in ^{16}O [35] and we confirmed that the dependence was quite small around $\hbar\Omega = 14\text{MeV}$ if perturbation corrections were included. As for the nucleon single-particle potential u_N , we used the fixed data [22] calculated using the Paris potential [27], and we have assumed that $u_\Sigma = 0$. The boundary numbers ρ_2 , ρ_F and ρ_X are taken as $\rho_2 = 1$, $\rho_F = 1$ and $\rho_X = 12$ for $^{17}_\Lambda\text{O}$.

We present, in Fig. 4, the ρ_1 dependence of the single-particle energies of Λ including the perturbation corrections shown in Fig. 3. The parameter ρ_1 specifies the $P_{\Lambda N}^{(1)}$ space in which the ΛN effective interaction $\tilde{v}_{\Lambda N}^{(2)}$ is determined rigorously without any approximation as discussed in Sec. 3. Therefore, as ρ_1 becomes larger, the calculated result does more accurate. If the result converges at a certain number ρ_1 , we may say that the method of the two-step calculation of the ΛN effective interaction would be trustworthy. As shown in Fig. 4, the results are convergent and stable for $\rho_1 \geq 7$ for each YN potential. If we take ρ_1 larger than 6, our model space is considered to be sufficiently large. We note that the model space with $\rho_1 = 6$ corresponds to that used in the G -matrix calculations made by Halderson [16] and by Hao and Kuo [17,21].

Table 1 shows the dependence of typical matrix elements of the ΛN effective interaction at $\rho_1 = 8$ on the value N_{max} which denotes the number of the h.o. basis functions. The number N_{max} determines the dimension of the space of relative states in the first-step calculation. When we solve the eigenvalue equation (3.19), taking into account the tensor and ΣN - ΛN couplings, we need $2 \times 2 \times N_{\text{max}}$ basis functions in each partial-wave channel. Table 1 shows that the results are convergent if we take 120 for the $J\tilde{A}$ and $J\tilde{B}$ potentials, and

200 for the NSC potential as N_{\max} . We have also made sure that the situation of convergence is similar in the other matrix elements of the ΛN effective interaction. As we employ the NSC potential given in the configuration-space representation, we need much more basis states than in the Jülich potential given in the momentum-space representation in order to treat short-range correlations accurately.

Table 1

Dependence of typical diagonal matrix elements $\langle ab|\tilde{v}_{\Lambda N}^{(2)}|ab\rangle_J$ on N_{\max} in units of MeV, where a and b denote the Λ and N single-particle orbits, respectively, labeled as $1 = 0s_{1/2}$, $2 = 0p_{1/2}$ and $3 = 0p_{3/2}$, and J is the total angular momentum.

N_{\max}	$a = 2, \quad b = 2, \quad J = 1$			$a = 3, \quad b = 2, \quad J = 2$		
	$\tilde{J}\tilde{A}$	$\tilde{J}\tilde{B}$	NSC	$\tilde{J}\tilde{A}$	$\tilde{J}\tilde{B}$	NSC
40	-0.991	-1.331	-0.347	-0.825	-0.828	-1.146
60	-1.100	-1.432	-0.402	-0.909	-0.892	-1.186
80	-1.145	-1.483	-0.408	-0.932	-0.920	-1.191
100	-1.152	-1.492	-0.412	-0.937	-0.926	-1.195
120	-1.152	-1.492	-0.417	-0.937	-0.925	-1.198
140			-0.422			-1.201
160			-0.426			-1.204
180			-0.428			-1.205
200			-0.428			-1.205

Table 2

Diagonal matrix elements $\langle ab|\tilde{v}_{\Lambda N}^{(2)}|ab\rangle_J$ in units of MeV. The notations are the same in Table 1.

a	b	J	$\tilde{J}\tilde{A}$	$\tilde{J}\tilde{B}$	NSC	a	b	J	$\tilde{J}\tilde{A}$	$\tilde{J}\tilde{B}$	NSC
1	1	0	-1.31	-0.06	-4.04	2	3	1	-0.38	-0.75	-0.42
1	1	1	-2.49	-3.47	-1.38	2	3	2	-0.94	-0.92	-1.26
1	2	0	-0.78	-1.22	-0.39	3	1	1	-0.94	-0.62	-1.45
1	2	1	-0.95	-0.98	-0.70	3	1	2	-1.38	-1.80	-1.06
1	3	1	-0.46	-0.11	-1.13	3	2	1	-0.50	-0.90	-0.34
1	3	2	-1.17	-1.50	-1.00	3	2	2	-0.94	-0.93	-1.21
2	1	0	-1.05	-1.56	-0.54	3	3	0	-0.49	0.65	-2.03
2	1	1	-0.84	-0.84	-0.95	3	3	1	-0.73	-1.16	-0.02
2	2	0	-0.10	0.57	-0.71	3	3	2	-0.20	0.12	-0.96
2	2	1	-1.15	-1.49	-0.43	3	3	3	-1.34	-1.78	-0.84

We show, in Table 2, the diagonal matrix elements of the $0s$ - $0p$ shell ΛN effective interaction calculated in the second-step procedure. Because of the different characteristics of the original YN potentials, the results depend strongly on the YN potentials used in the calculation. Considering the situations of convergence shown in Fig. 4 and Table 1, we have used the numbers $\rho_1 = 8$, and $N_{\max} = 120$ and 200 for the Jülich and Nijmegen potentials, respectively, in Table 2. We shall use hereafter the same set of the numbers in the following tables.

In the present calculation we consider three kinds of the treatments of the Pauli-blocking effect in the first-step procedure.

- (1) The Pauli blocking in the spaces $Q_{\Lambda N}^{(x)}$ and $Q_{\Sigma N}^{(x)}$ is not considered, that is, the original YN interaction v_{YN} is used in place of the interaction \bar{v}_{YN} in Eq. (3.10).
- (2) The Pauli blocking in both the spaces $Q_{\Lambda N}^{(x)}$ and $Q_{\Sigma N}^{(x)}$ is considered, that is, we take only the first term of \bar{v}_{YN} in Eq. (3.10).
- (3) The Pauli blocking in the spaces $Q_{\Lambda N}^{(x)}$ and $Q_{\Sigma N}^{(x)}$ is considered, but the diagonal interaction, the second term of \bar{v}_{YN} in Eq. (3.10), is restored.

It is noted that, in the above three cases, the predominant Pauli-blocking effect which comes from the low-momentum ΛN states contained in the $P_{\Lambda N}^{(x)}$ space is taken into account rigorously in the second-step calculation.

We here discuss the dependence of the different treatments of the Pauli-blocking effect in the first-step procedure for three YN interactions. In Table 3 we display the single-particle energies of Λ including the perturbation corrections shown in Fig. 3. Rows (1), (2) and (3) correspond to different treatments (1), (2) and (3) of the Pauli-blocking effect in the first-step procedure. All the single-particle energies of Λ in the $0s_{1/2}$, $0p_{3/2}$ and $0p_{1/2}$ states in row (1) are negative for three kinds of the YN interactions, and the results for the J \tilde{B} and NSC potentials have almost the same order of magnitude. The results

Table 3

Dependence of the single-particle energies of Λ on the different treatments of the Pauli-blocking effects for the $0s_{1/2}$, $0p_{3/2}$ and $0p_{1/2}$ states including the perturbation corrections.

Pauli	$0s_{1/2}$			$0p_{3/2}$			$0p_{1/2}$		
	J \tilde{A}	J \tilde{B}	NSC	J \tilde{A}	J \tilde{B}	NSC	J \tilde{A}	J \tilde{B}	NSC
(1)	-14.09	-16.47	-16.74	-2.49	-3.95	-4.47	-1.29	-2.46	-3.07
(2)	-10.96	-13.42	-9.23	-0.16	-1.76	-0.04	1.28	-0.17	1.68
(3)	-12.19	-15.54	-11.69	-0.78	-2.81	-1.17	0.64	-1.24	0.49

in row (2) show that all the single-particle energies are less attractive than those given in row (1), and especially, those for the $0p_{1/2}$ state are positive (unbound) for the $\tilde{J}\tilde{A}$ and NSC potentials. This means that the Pauli-blocking effect contributes repulsively to the single-particle energies and plays a very significant role although it depends on the YN potential used. In row (3) the single-particle energies become attractive in comparison with those given in row (2) because of the restoration of the “diagonal” interaction. The results in row (3) for the $\tilde{J}\tilde{A}$ and NSC potentials have almost the same order of magnitude in contrast with those given in row (1). On the whole, we see from Table 3 that the Pauli-blocking effect is significantly large and yields different contributions to the single-particle energies dependently on the YN potentials. The difference between the results shown in rows (2) and (3) is fairly large. Therefore, we must treat the Pauli-blocking effect carefully and should adopt the treatment of case (3) as the most accurate one. In Fig. 4, Table 1 and Table 2, we have displayed the results of the treatment of the Pauli-blocking effect in case (3).

Next, the partial-wave contributions to the first-order potential energy of Λ in the $0s_{1/2}$ state are given for the different treatments of the Pauli-blocking effect. We see from Table 4 that the contributions of the 1S_0 and 3S_1 - 3D_1 channels are very different dependently on the YN interactions employed. In the 1S_0 channel the largest Pauli-blocking effect is brought about in the result for the $\tilde{J}\tilde{A}$ potential. On the other hand, in the 3S_1 - 3D_1 channel we have an extremely large Pauli-blocking effect for the NSC potential. This result shows that the NSC potential has the strong ΣN - ΛN coupling in the 3S_1 - 3D_1 channel as discussed in Ref. 20). The absolute values contributing to the first-order potential energy of Λ in the channels with angular momenta $l \geq 1$ are much

Table 4

Partial wave contributions to the first-order potential energy of Λ in the $0s_{1/2}$ state. All entries are in MeV.

		1S_0	3S_1 - 3D_1	3P_0	1P_1 - 3P_1	3P_2 - 3F_2	Total
$\tilde{J}\tilde{A}$	(1)	-4.87	-17.94	0.32	0.91	-0.10	-21.67
	(2)	-2.51	-17.26	0.40	1.14	-0.09	-18.32
	(3)	-3.14	-17.82	0.37	1.07	-0.10	-19.61
$\tilde{J}\tilde{B}$	(1)	-0.24	-24.81	0.31	1.43	0.31	-23.01
	(2)	0.12	-22.61	0.37	1.59	0.32	-20.22
	(3)	-0.17	-24.43	0.35	1.54	0.31	-22.40
NSC	(1)	-9.70	-14.16	0.18	1.80	-1.86	-23.73
	(2)	-9.58	-7.29	0.28	2.05	-1.84	-16.38
	(3)	-9.59	-9.79	0.24	1.96	-1.85	-19.04

smaller than those in the 1S_0 and 3S_1 - 3D_1 channels.

In Table 5 we display the single-particle energies of Λ with the contributions of the perturbation corrections given in Fig. 3, where the Pauli-blocking effect is treated accordingly to case (3). From this table we see that the $J\tilde{A}$ and NSC

Table 5

Contributions to the single-particle energies of Λ in units of MeV. Here KE and PE stand for the kinetic energy and the first-order potential energy, respectively. Rows (a), (b) and (c) are the contributions of diagrams (a), (b) and (c) in Fig. 2, respectively. The quantity $\Delta\varepsilon_{ls}$ is the spin-orbit splitting of the $0p$ states.

$J\tilde{A}$				
	$0s_{1/2}$	$0p_{3/2}$	$0p_{1/2}$	$\Delta\varepsilon_{ls}$
KE	10.50	17.50	17.50	
PE	-19.62	-14.50	-12.97	1.53
(a)	-1.73	-2.83	-3.03	-0.20
(b)	-1.00	-0.43	-0.05	0.39
(c)	-0.33	-0.51	-0.81	-0.31
Total	-12.19	-0.78	0.64	1.41
$J\tilde{B}$				
	$0s_{1/2}$	$0p_{3/2}$	$0p_{1/2}$	$\Delta\varepsilon_{ls}$
KE	10.50	17.50	17.50	
PE	-22.44	-16.43	-14.81	1.62
(a)	-1.39	-2.19	-2.36	-0.17
(b)	-1.31	-0.51	-0.11	0.40
(c)	-0.90	-1.17	-1.45	-0.28
Total	-15.54	-2.81	-1.24	1.57
NSC				
	$0s_{1/2}$	$0p_{3/2}$	$0p_{1/2}$	$\Delta\varepsilon_{ls}$
KE	10.50	17.50	17.50	
PE	-19.06	-14.71	-12.95	1.76
(a)	-1.68	-2.76	-3.04	-0.28
(b)	-0.93	-0.52	-0.12	0.41
(c)	-0.52	-0.68	-0.90	-0.22
Total	-11.69	-1.17	0.49	1.66

potentials yield almost the same values of the first-order potential energy PE and the perturbation corrections in spite of having different partial-wave contributions as shown in Table 4. The single-particle energies obtained for the $J\tilde{B}$ potential are most attractive among the YN interactions used. It should be noted here that the values of the KE plus PE in the $0p$ states for three kinds of YN interactions are positive (unbound). Therefore, the perturbation corrections play a decisive role when we argue that the Λ in the $0p$ states is bound or not. Correction (a) in Fig. 3 is induced by the presence of non-diagonal terms in the one-body hamiltonian $h_A = t_A + u_A$. This effect should be treated in the Hartree-Fock (HF) calculation. However, in the present calculation, we treat this effect perturbatively. These non-diagonal terms work to change the single-particle wave function of Λ . Therefore, this procedure is indispensable in this approach since we have taken a fixed value of $\hbar\Omega$ commonly for Λ and N . This correction (a) contributes most attractively to the single-particle energies among all the corrections in Fig. 3. The magnitude of every correction is considerably small in comparison with the first-order potential energies for all the YN interactions used. All the perturbation corrections contribute, however, attractively to the single-particle energies of Λ , and do not change the mutual spacings of the single-particle states significantly. This trend is very different from the situation observed in ordinary nuclei such as ^{16}O and ^{40}Ca , in which the correction terms contribute to about 40% of the calculated spin-orbit splitting of the $0p$ or $0d$ states [22,23]. The role of many-body correlations, such as the core polarization, does not seem to be important in the spin-orbit splitting of Λ .

5 Concluding remarks

We have presented a formulation of the UMOA to apply it to a structure calculation of hypernuclei. We have introduced a unitary transformation $\exp(S)$, with the correlation operator S , that describes two-body correlation of hyperon and nucleon. An equation has been given for determining the correlation operator S as the equation of decoupling so that the unitarily transformed interaction does not have non-zero matrix elements between the model space P of low-lying ΛN states and its complement Q . The effective interaction has been given by the projected interaction onto the P space. The effective interaction thus determined has properties of being hermitian, energy-independent and decoupled between the P and Q spaces. In comparison with the usual G -matrix approach, the use of such an effective interaction would have some advantages in describing many-body systems.

The UMOA has been applied to the calculation of the structure of $^{17}_{\Lambda}\text{O}$. The ΛN effective interaction has been calculated by employing three YN potentials, namely, the $J\tilde{A}$, $J\tilde{B}$ and NSC potentials. With the ΛN effective inter-

action, we have calculated the single-particle energies of Λ in the $0s_{1/2}$, $0p_{3/2}$ and $0p_{1/2}$ states.

We have seen that how to treat the Pauli-blocking effect is very important, especially when we solve the problem of the ΣN - ΛN coupling. In the usual treatment one excludes all the interactions acting in a space of ΛN and ΣN states in which a nucleon is in occupied states. However, this treatment of the Pauli-blocking effect would lead to overmuch counting. We have introduced a new treatment in which, while we exclude the interactions acting among forbidden YN states as in the usual treatment, we restore the interactions that are diagonal in occupied nucleon states. We have observed that this treatment of the Pauli-blocking effect has given rise to a significant effect as compared with the usual treatment.

The present results have shown that Λ in the $0p_{1/2}$ state is unbound for the $\tilde{J}\tilde{A}$ and NSC potentials and, on the other hand, bound for the $\tilde{J}\tilde{B}$ potential. The spin-orbit splitting of the single-particle levels of Λ in the $0p$ states has been given by 1.41, 1.57 and 1.66 for the $\tilde{J}\tilde{A}$, $\tilde{J}\tilde{B}$ and NSC potentials, respectively. These values are very large compared with the results in Ref. [19]. It will be of high interest for the single-particle nature of Λ in $^{17}_{\Lambda}\text{O}$ to be established experimentally.

In the application of the UMOA some problems have still remained. The unitarily transformed hamiltonian [22] contains originally three-or-more-body interactions. The effects of these many-body interactions have been evaluated in the previous works of calculating nuclear properties, and we have concluded that the contributions of these many-body terms are much smaller than those of two-body interactions but sizable in some cases. We therefore should estimate the effects of these many-body interactions also in hypernuclei.

Another refinement to be made is that, in the present calculation, the single-particle energies of Σ in intermediate states are neglected. However, the single-particle spectrum of Σ has a possibility of giving rise to some contributions to the ΛN effective interaction. Although the properties of the single-particle energies of Σ have not always been made clear, we should investigate how much the spectrum of Σ affects in the determination of the ΛN effective interaction. This problem is also a remaining task in the UMOA.

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